

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2060B Mathematical Analysis II (Spring 2017)
Tutorial 11

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1. (a) State Weierstrass' M-test.
- (b) A Weierstrass function is a continuous function that is nowhere differentiable. A typical example is given by:

$$f(x) := \sum_{n=0}^{\infty} 0.9^n \cos(7^n \pi x), x \in \mathbb{R}$$

Show that the above function is well defined and is continuous on \mathbb{R} .

2. (Power Series)

- (a) State the Hadamard's formula, explain and define radius of convergence of a power series.
- (b) Find the radii of convergence of the following power series. Moreover, prove whether they converge at boundary points.

- i. $\sum_{n=0}^{\infty} x^n$.
- ii. $\sum_{n=0}^{\infty} nx^n$.
- iii. $\sum_{n=0}^{\infty} n!x^n$.
- iv. $\sum_{n=1}^{\infty} \frac{1}{n^2}x^n$.

3. (Taylor Series)

Compute the Taylor series centred at 0 for the following functions, find their radii of convergence, and prove whether they converge at boundary points.

- (a) $f(x) = e^x$.
- (b) $f(x) = \frac{1}{1-x}$.
- (c) $f(x) = \ln(1-x)$.
- (d) $f(x) = \frac{1}{1+x^2}$.
- (e)

$$f(x) = \begin{cases} \exp(-\frac{1}{x^2}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

You do not need to compute the exact values of the derivatives of f .